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A computer simulation of the time dependence of the erosion of Pyrex glass by glass beads

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With the aid of scanning electron microscopy (SEM) observations of the surface of Pyrex glass, eroded by spherical glass beads, a recursive algorithm has been formulated which predicts the weight-loss from the specimen as a function of time. This method takes into account three mechanisms of material removal: material chipped off when cone cracks form on the surface; materials removed due to interaction of cone cracks on the flat surface; and materials removed due to chipping of exposed cone frustrums and the underlying rough surface. Multi-particle erosion tests were performed on the Pyrex glass target material employing spherical glass beads as impacting particles. Four different particle velocities and average particle sizes were employed as the varaibles in the experiments. An analysis of the experimental results in conjunction with the mathematical model showed a very good agreement between experimental results and theoretical prediction.

1. Introduction

Erosion of materials has become recognized as a serious problem in a wide variety of engineering applications. Coal-conversion plants, turbine blades and helicoptor rotor blades are but a few examples in which material deterioration by erosion is a very serious problem. This has generated a considerable amount of interest directed at being able to predict the erosion of materials in service environments as well as to be able to understand the nature of the erosion in a more fundamental manner. The objectives of this programme were designed specifically to study the fundamental mechanisms of erosion. For this purpose ideal and simple experimental conditions were selected which could directly be treated for analytical and theoretical calculations. Since the fracture mechanism of brittle materials is better understood than ductile materials, it was decided that a brittle material with isotropic properties should be employed as the target material in this work. For these reasons Corning 7740 Pyrex glass was chosen as the target material to be eroded. In order to keep a simple and well defined geometry of the impacting particles, spherical glass beads of diameters ranging from 380 to 760 μm were employed as the impacting particles. A typical test consisted of monitoring the weight-loss of a Pyrex glass specimen during erosion under certain specific conditions by the glass beads.

The singular event in such a process of erosion is the impact of one spherical glass bead on the flat surface of Pyrex glass. During this process when a spherical hard particle is indenting a nominally brittle material, the target material may deform either in an elastic–plastic manner or in an elastic–brittle manner, depending upon the load and the particle size. For particles of a size greater than a critical value, an elastic–brittle fracture will occur first which is of a type commonly described as Hertzian fracture [1]. In the Hertzian fracture test, a hard spherical ball is pressed against a nominally brittle target material until a ring crack appears near the area of contact. On further increasing the load, this ring crack develops into a cone crack. Several investigators have studied the Hertzian fracture test (for a Review see [2]) and consequently expressions are available which relate the cone-crack dimensions to the load applied and to the indentor size.

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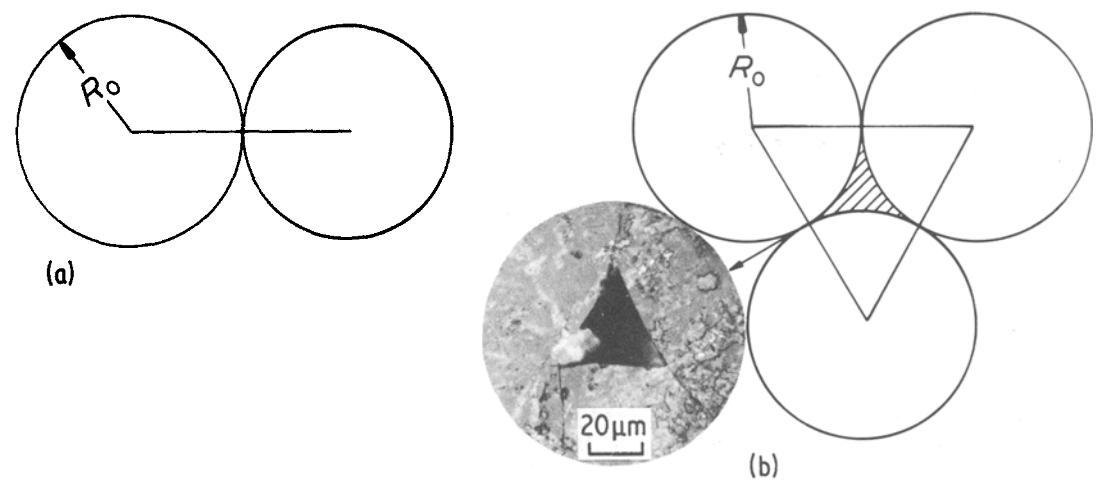

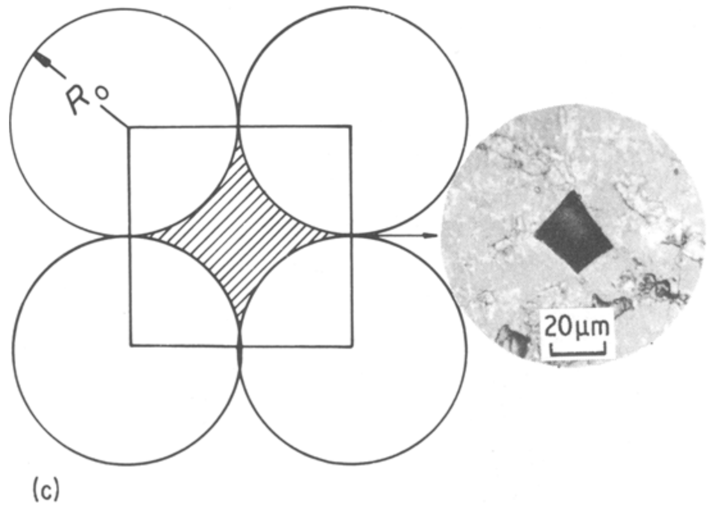

Figure 1 Interaction configurations for the cone-cracks. Doublet, triplet and quadruplet arrangements are illustrated, respectively, in (a), (b) and (c).

It has been shown [3, 4] that erosion of a brittle solid can occur by the formation and interaction of Hertzian cone-cracks. Three or more cone-cracks must form within given distances of each other to remove material. Fig. 1 illustrates the configuration of a doublet, triplet and a quadruplet. The maximum volume that can be removed by such interactions of cone-cracks can be calculated from purely geometrical considerations, as a function of particle velocity and size [4]. Adler [3] has computed the probability of forming a triplet when a given number of particles strike the surface and has forwarded an expression for the total number of triplets, T, existing on the surface at any time, n:

$$T(n) = \frac{N(N-1)(N-2)}{3!}\frac{A_r}{A}\frac{(r_2^2-r_1^2)}{A}, \quad (1)$$

where N is the number of particles striking in time n, A is the area of the specimen exposed, $r_2 - r_1$ is the radial distance between two cone-cracks in order to interact and A_r is the area in which a third ring crack forms a triplet.

If the volume removed by each triplet remains constant, Equation 1 suggests erosion to be proportional to the third power of time. However, the experimental data [3, 4] indicates that the weight-loss varies linearly with time after an initial transient stage. This difference between the theoretical prediction and experimental results has prompted the present work in which an attempt has been made first to understand the physical process and then express it in terms of mathematical equations.

2. Theory

Fig. 2 shows the SEM photomicrographs of the eroded surface of Pyrex glass by spherical glass beads. After a short time of erosion when the surface is still largely flat, material is only removed by the formation and intersection of cone-cracks

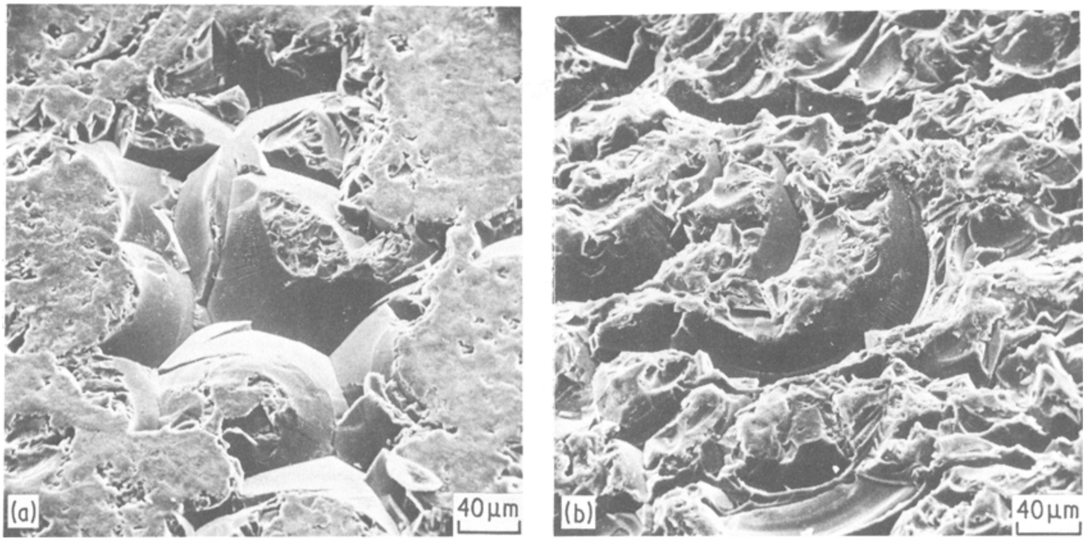

Figure 2 SEM micrographs of Corning 7740 pyrex glass surface after erosion with Sample 203 ($D = 760 \pm 20\,\mu m$) glass beads at an incident angle of 90° and an average particle velocity of $78\,m\,sec^{-1}$ (a) after 10 sec (b) after 100 sec.

when particles impact on the surface. However, at long times, when the surface is relatively rough, material removal occurs by chipping only. Clearly, these two mechanisms, namely the interaction of cone-cracks and chipping, must be considered in developing a satisfactory model for the process. Other mechanisms of material removal or strength degradation such as lateral cracks and radial cracks were not observed to occur under the experimental conditions of the present investigation. Lateral and radial cracking has been observed to occur under suitable conditions of high particle velocity and appropriate material parameters [5]. Therefore, it is important to identify the dominant modes of material removal before attempting to model the process.

Fig. 3 presents a schematic composite plot of the weight-loss as a function of time where the weight-loss, $W_T(n)$, at any time, n, is given by:

$$W_T(n) = W_0 + W(n) + W_f(n), \quad (2)$$

where W_0 is the weight-loss associated with the formation of the ring cracks. Its value is normally small and will be determined empirically. $W(n)$ is the weight-loss associated with the interaction of the cone-cracks on the flat surface in time n sec and $W_f(n)$ is the weight-loss due to the chipping of the cone frustrums and the uneven substrate in n seconds. At the beginning of the nth second, let the area, which is no longer flat, be given by $A_0(n-1)$ and the number of cone-cracks interacting during the nth second be $N(n)$. Assuming that material removal occurs only by the formation of triplets and that three distinct cone-cracks interact to form a triplet, one can express $A_0(n)$, the rough area at the end of the nth sec as follows:

$$A_0(n) = A_0(n-1) + \frac{N(n)}{3} A_t, \quad (3)$$

where A_t is the flat area associated with each triplet. The value of $N(n)$ can be given by

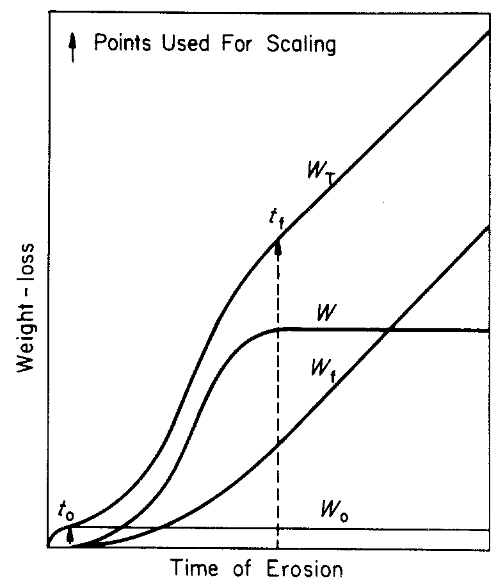

Figure 3 Schematic composite plot of the weight-loss as a function of time of erosion.

TABLE I Particle velocity and size employed for erosion in the present study

Particle velocity ($m\ sec^{-1}$)	Particle diameter, D (μm)			
	Sample 203 $D = 760 \pm 20$	Sample 253.5 $D = 680 \pm 40$	Sample 304 $D = 490 \pm 40$	Sample 405 $D = 380 \pm 40$
112	×			
78	×	×	×	×
55	×			
35	×			

$$N(n) = nF[A - A_0(n-1)], \quad (4)$$

where F is the number of particles striking per unit area and time and A is the total area exposed. If V is the volume of the material dislodged from each triplet, one can express $W(n)$ as follows:

$$W(n) = \frac{dA_0(n)V}{A_t}, \quad (5)$$

where d is the density of the target material. At short times most of the surface is flat but there are fewer cone-cracks on the surface. Consequently, the rate of material removal by the interaction of cone-cracks will be low, as shown in Fig. 3. At long erosion times, most of the surface will be rough so that the bulk of the material occurs by chipping and the rate of material removed by cone-crack interaction is again low. At a time between these two stages, the rate of material removal by cone-crack interaction is maximized.

On the other hand, chipping occurs mainly on an uneven surface, and therefore its rate is almost zero in the beginning when most of the surface is flat. As material removal occurs by cone-crack interaction, more and more surface becomes rough and more material is then removed by chipping. Thus, the rate of material removal by chipping continued to increase until it reaches a constant or steady-state value when all of the exposed surface has become rough or uneven. If K' is the steady-state rate of material removal by chipping and $W_f(n-1)$ is the weight of material chipped off in $(n-1)$ seconds, $W_f(n)$ can be given by:

$$W_f(n) = W_f(n-1) + A_0(n)K'F. \quad (6)$$

Combining Equations 3 to 6, a recursive algorithm can be formulated which calculated the weight-loss by cone-crack interaction, W, weight-loss by chipping, W_f, and the total weight-loss, W_T at every second. This scheme can be employed as long as A_0 is less than A. When A_0 is greater than A, all the surface is rough and all the material is being removed by chipping alone, as expressed by Equation 6 with A_0 equal to A. During this stage, W will remain constant and is given by its value when A_0 just became equal to A, the total surface area. Fig. 3 shows a schematic plot of the output obtained from this recursive scheme. The time, t_0, in Fig. 3 is the incubation period after which cone-cracks start interacting. The time t_f is one when the whole surface of the target becomes uneven ($A_0 = A$) and material loss can occur by chipping alone.

A method for calculating the volume, V, of the material removed by a triplet has been presented in an earlier paper [4]. This method, however, gives the maximum possible amount of the volume removed for given size of the cone-crack. As shown by Adler [3], there is always a probability term associated with the formation of the triplet so that the size of the triplet and the volume removed by a triplet are always smaller than their maximum possible values. This consideration can be incorporated in the present scheme by assuming an effective flux of particles, F_e, in place of the true flux, F, for the process of material removal by cone-crack interaction. This effective flux can be estimated by constraining the value of $(t_f - t_0)$ in the computer program to its experimental value and also by the knowledge of the true flux, F, and the probability formation of a triplet, P_t:

$$F_e = P_t F. \quad (7)$$

Incidently, this will also provide us with a check for the internal consistency of the scheme by comparing the values of the effective flux obtained by the two methods.

3. Results and discussion

Multi-particle erosion tests were conducted on Pyrex glass by spherical glass beads at normal angle of impingement. The various particle velocities and particle sizes employed in the tests are listed in Table I. Particle velocity was measured by the two disc methods described by Ruff and

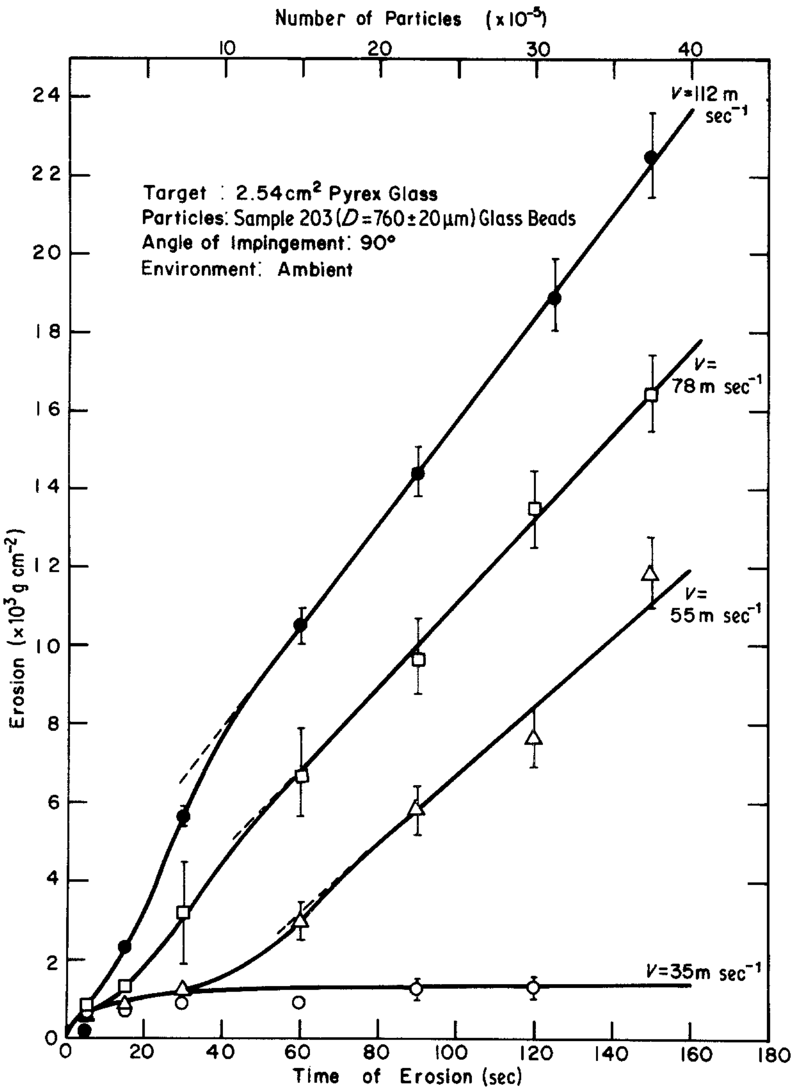

Figure 4 Erosion ($g\,cm^{-2}$) plotted as a function of time and the number of particles at various particle velocities.

Ives [6] and the particle size was taken as the graphical mean of the particle-size frequency distribution. Erosion tests were carried out by bombarding a target specimen with a stream of particles carried by air for a given length of time. The weight-loss of the specimen was plotted as a function of time and number of particles striking. Fig. 4 shows such a plot for four different particle velocities while Figs 5 and 6 illustrate these curves for four different particle sizes. All of these curves have the characteristic form first noticed by Adler [3] and schematically represented in Fig. 3.

In order to employ the algorithm presented in the previous section, the volume removed, V, and the flat surface area, A_t, of a triplet were estimated by the method outlined in an earlier paper [4]. The steady-state erosion rate, K', was determined empirically from measuring the slope of the steady-state part of the curves presented in Figs 4 to 6. The computer program was then run with the necessary information so that the predicted value of $(t_f - t_0)$ matched with its experimental value by adjusting the value of effective flux, F_e. These values are listed in Table II for all the experimental conditions investigated in this work. In order to determine the deviation of predicted values of weight-loss from their experimental values, a scaling factor was computed as a ratio of the experimental and theoretical values of the weight removed at a time $(t_f - t_0)$ as a result of cone-crack interaction:

$$\text{Scaling factor} = \frac{(W_E - W_0 - W_f)}{W_{TH}}, \qquad (8)$$

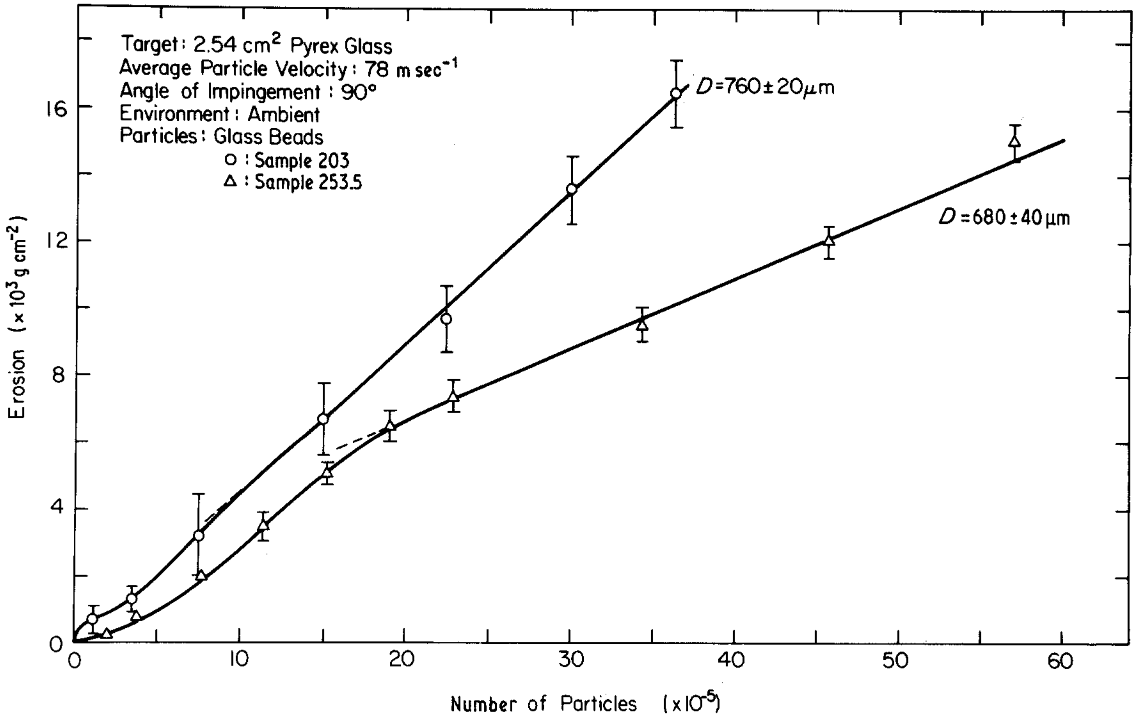

Figure 5 Erosion ($g\,cm^{-2}$) plotted as a function of the number of particles for two particles sizes, Sample 203 ($D = 760 \pm 20\,\mu m$) and Sample 253.5 ($D = 680 \pm 40\,\mu m$).

where W_E is the experimental value of the weight-loss at time t_f, W_0 is as shown in Fig. 3, W_f is the theoretical weight removed by chipping at time $(t_0 - t_f)$ and W_{TH} is the theoretical weight removed by cone-crack interaction at time $(t_0 - t_f)$. The values of the scaling factor for all the experimental conditions employed in this work are within an order of magnitude of unity and are presented in Table II. The computer program was then run with the appropriate values including

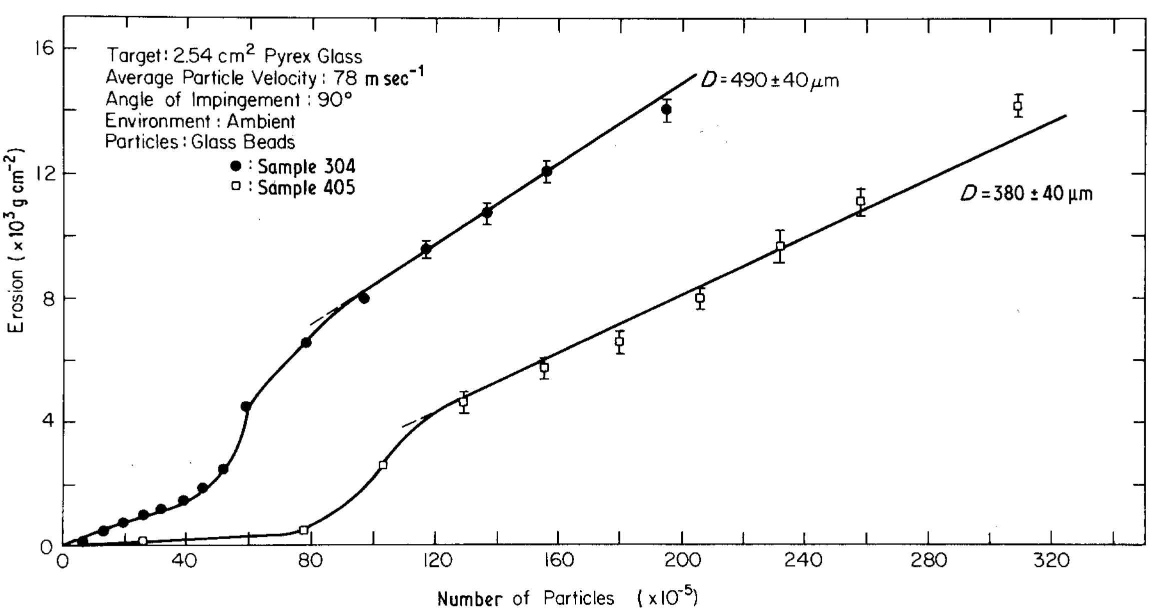

Figure 6 Erosion ($g\,cm^{-2}$) plotted as a function of the number of particles for two particles sizes, Sample 304 ($D = 490 \pm 40\,\mu m$) and Sample 405 ($D = 380 \pm 40\,\mu m$).

TABLE II Values of effective flux and scaling factor

Particle velocity (m sec^{-1})	Effective flux, F_e				Scaling factor			
	Sample 203 $D = 760 \pm 20$ (μm)	Sample 253.5 $D = 680 \pm 40$ (μm)	Sample 304 $D = 490 \pm 40$ (μm)	Sample 405 $D = 380 \pm 40$ (μm)	Sample 203 $D = 760 \pm 20$ (μm)	Sample 253.5 $D = 680 \pm 40$ (μm)	Sample 304 $D = 490 \pm 40$ (μm)	Sample 405 $D = 380 \pm 40$ (μm)
112	61				0.35			
78	48	58	60	1370	0.23	0.43	0.56	0.36
55	44				0.09			

TABLE III Comparison of calculated and curve-fitted values of effective particle flux

Particle velocity (m sec^{-1})	Calculated effective particle flux				Curve-fitted effective particle flux			
	Sample 203 $D = 760 \pm 20$ (μm)	Sample 253.5 $D = 680 \pm 40$ (μm)	Sample 304 $D = 490 \pm 40$ (μm)	Sample 405 $D = 380 \pm 40$ (μm)	Sample 203 $D = 760 \pm 20$ (μm)	Sample 253.5 $D = 680 \pm 40$ (μm)	Sample 304 $D = 490 \pm 40$ (μm)	Sample 405 $D = 380 \pm 40$ (μm)
112	110				61			
78	63	86	86	238	48	58	60	1370
55	53				44			

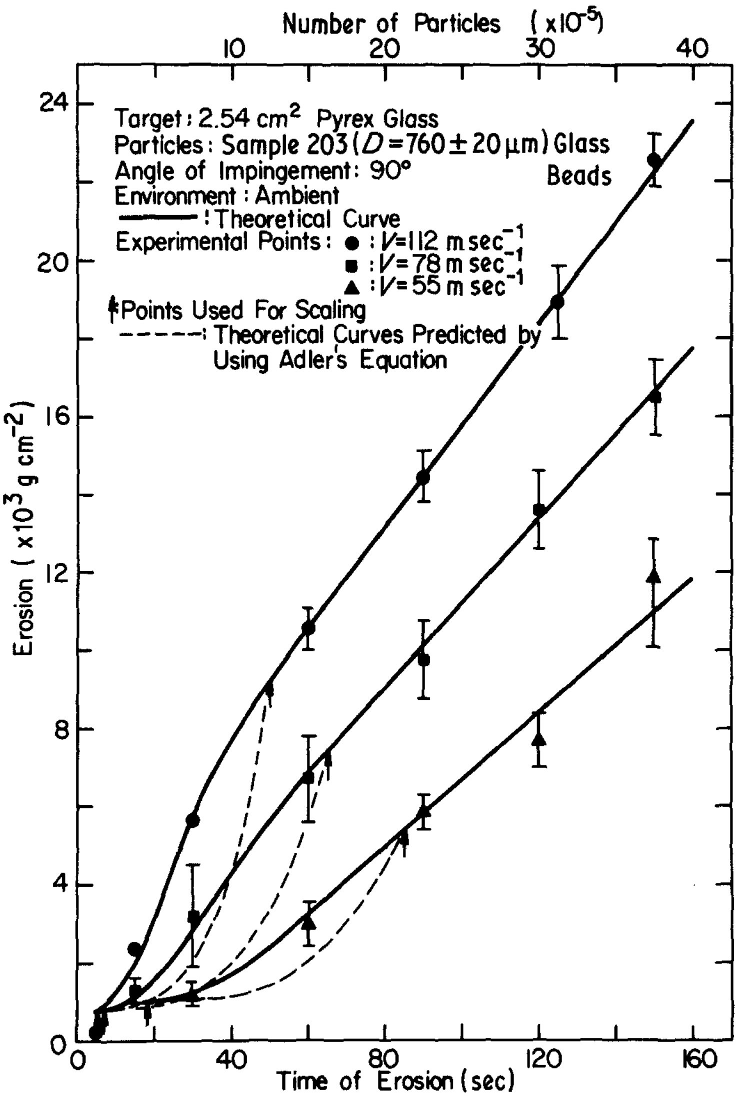

Figure 7 Comparison of the experimental values of erosion (g cm^{-2}) as a function of time and number of particles at various particle velocities with the predicted values of erosion (g cm^{-2}) by the present method and by Adler's method.

those presented in Table II and the resulting data has been plotted in Figs 7 to 9. Fig. 7 illustrates the comparison between the theoretical and experimental values of erosion as a function of time and the number of particles striking for three different particle velocities. Figs 8 and 9 show this comparison for four different particle sizes. In all these cases, a very good agreement between theoretical and experimental values of erosion is obtained.

In order to compare the results of this work with the theoretical prediction of Adler's equation [1], the same general approach, as outlined in the preceding section was employed, except Equations 3 and 4 were modified to incorporate Adler's equation, Equation 1. Thus, the rough surface area at any time can be given by:

$$A_0(n) = T(n)A_t, \tag{9}$$

where $T(n)$ is given by Equation 1 and N can be expressed as follows:

$$N = F_e n. \tag{10}$$

A recursive algorithm can again be formulated employing Equations 1, 2, 5, 6, 9 and 10 and its results can be compared with those of the present work and the experimental results. In the preceding scheme, the value of the effective flux, F_e, can be directly computed as follows:

$$F_e \simeq \frac{1}{t_f - t_0}\left[\frac{6}{A_t A_r \pi (r_2^2 - r_1^2)}\right]^{1/3}. \tag{11}$$

The computer program was constrained to match the experimental and theoretical curves at two points, $(t_f - t_0)$ time apart by the scaling factor

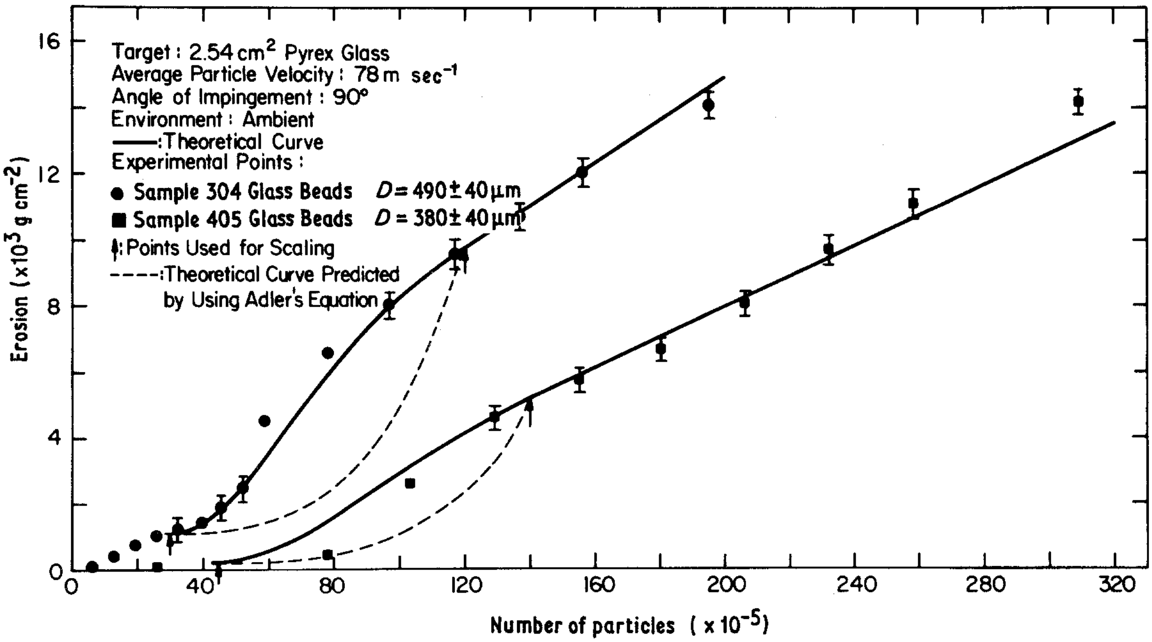

Figure 8 Comparison of the experimental values of erosion (g cm^{-2}) as a function of the number of particles for various particle sizes with the predicted values of erosion (g cm^{-2}) given by the present method and by Adler's method for Sample 253.5 ($D = 680 \pm 40\,\mu$m) and for Sample 253.5 ($D = 680 \pm 40\,\mu$m).

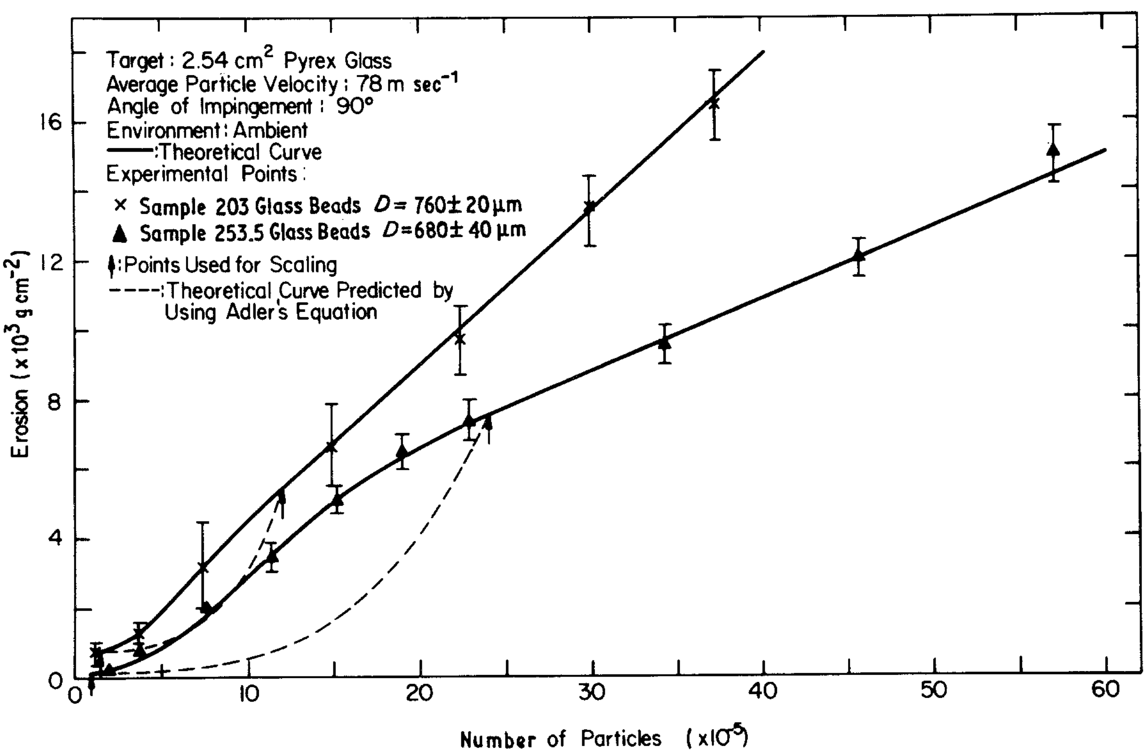

Figure 9 Comparison of the experimental values of erosion (g cm^{-2}) as a function of the number of particles for various particle sizes with the predicted values of erosion (g cm^{-2}) given by the present method and by Adler's method for Sample 304 ($D = 490 \pm 40\,\mu$m) and for Sample 405 ($D = 380 \pm 40\,\mu$m).

of Equation 8. The output obtained from the computer program, after providing it with appropriate values of the variables was plotted in Figs 7 to 9 for all the conditions of erosion investigated in this work. A comparison between the theoretical curves predicted by using Adler's equation and those obtained by employing the method outlined in the present work indicates that the latter offers a much better fit to the experimental points.

As indicated earlier, in order to check the internal consistency of the present theoretical calculations, the value of effective flux, F_e, can also be estimated by Equation 7 if the probability to form a triplet, P_t, is known. This probability was estimated employing the following equations:

$$P_t = \frac{V_E}{V_T}, \tag{12}$$

where V_E is the experimental value of the volume removed by three impacts and V_T is the theoretical value of the volume removed by a triplet. Table III lists the effective particle flux values obtained from the two methods and for the most part they agree with each other within a factor of 2. Though the two sets of values of effective particle flux agree with each other within a factor of 2, for the most part the calculated effective particle flux values are greater than the curve-fitted effective particle flux values. Since this difference increases with the particle velocity, a probable source of this deviation may be mechanisms of material removal other than cone-crack interaction and chipping [5].

4. Conclusion

A mathematical model has been developed which simulates the characteristic erosion against time curves when spherical glass beads impact on Corning 7740 Pyrex glass, which is a nominally brittle material. Two suitably located experimental points are needed to successfully apply this method for reproducing erosion against time curves. However, the qualitative agreement between the experimental and theoretical curves is very good and better than that obtained with the previously existing model.

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